

AN EXPERIMENTAL METHOD OF DESIGN ANALYSIS  
USING BRITTLE MATERIAL

BY  
JONATHAN ARNOLD BARKER

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J. A. Barker



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by

Jonathan Arnold Barker  
Lieutenant Commander, United States Navy

Submitted in partial fulfillment  
of the requirements  
for the degree of  
MASTER OF SCIENCE  
IN  
MECHANICAL ENGINEERING

United States Naval Postgraduate School  
Annapolis, Maryland  
1950

Thesis  
B22

This work is accepted as fulfilling  
the thesis requirements for the degree of

MASTER OF SCIENCE  
IN  
MECHANICAL ENGINEERING

from the  
United States Naval Postgraduate School

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Paul J. Kiefer  
Department of Mechanical Engineering

Approved:

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R. S. Glasgow  
Academic Dean

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## PREFACE

The investigation embodied in this thesis was carried out at the Argonne National Laboratory, Chicago, Illinois during the fall of 1948 and the spring of 1949. The work was done as a phase of the study of the behaviour of tubular structures.

The assistance of Messrs. R. O. Brittan and W. W. Galbreath of the Argonne National Laboratory, and Messrs. W. E. Berkey and P. Newhouse of the Westinghouse Electric Corporation is acknowledged with appreciation. Associate Professor G. H. Lee of the Postgraduate School was particularly helpful in the preparation of this thesis.





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## TABLE OF SYMBOLS AND ABBREVIATIONS

A	Cross sectional area perpendicular to dielectric flux, sq. in.
Btu	British thermal unit.
cc	Cubic Centimeters
cm	Centimeter
t	Thickness of specimen in direction parallel to flux, inches.
e"	Loss factor, product of dielectric constant and power factor.
E	Young's modulus.
f	Frequency, cycles per second.
°F	Degrees Fahrenheit.
ft	Foot.
sq ft	Square foot.
hr	Hour.
in	Inch.
sq in	Square inch.
k	Coefficient of thermal conductivity in Btu per hr-ft-°F.
K	Dielectric constant.
KW	Kilowatt.
(p.f.)	Power factor.
psi	Pounds per square inch.
P <sub>b</sub>	Rate of generation of heat required to cause failure, Btuper hour.
P <sub>w</sub>	Rate of generation of heat by dielectric heating, watts.
α	Coefficient of thermal expansion, in per in-°F.
ε	Strain.
σ	Ultimate strength, psi.



## ABSTRACT

An investigation has been conducted of an experimental approach to the strength design of tubular structures. The approach employed was the brittle material method.

The tubular structure to be studied was one having uniform internal generation of heat. Dielectric heating was found to be satisfactory for inducing the heating. Hydrocal Plaster of Paris with Aquadag and Antimony Trioxide added appeared to be a satisfactory brittle material.



# I

## AN EXPERIMENTAL METHOD OF STRENGTH DESIGN ANALYSIS USING BRITTLE MATERIAL

This investigation is an approach to the strength design of tubular structures in which heat is being uniformly internally generated. There is coolant flow through and around the tubular structure. The approach is by application of the brittle material method. The investigation may be divided into three parts:

1. Finding a means for the uniform internal generation of heat.
2. Finding a suitable material.
3. The conduction of tests.





## II

### THE USE OF A BRITTLE MATERIAL

The basis of this approach is to determine the rate of uniform internal generation of heat that will cause failure of the tubular sample. The technique of testing was to increase the rate of generation of heat in small steps, allowing sufficient time for steady state conditions to exist, until failure occurred.

It is desirable to use a brittle material in this approach. A brittle material is defined as a perfectly elastic material whose ultimate strength and proportional limit coincide.

The advantage lies in the manner in which brittle materials fail under stress. The brittle type failure gives a distinct audible and visual indication that the thermal stresses in the material have exceeded its ultimate strength.

The tubular structure chosen for investigation was a right circular cylinder four inches long, one inch in inner diameter and 1.25 inches outer diameter.



### III

#### METHODS THAT MAY BE EMPLOYED TO GENERATE UNIFORM INTERNAL HEAT

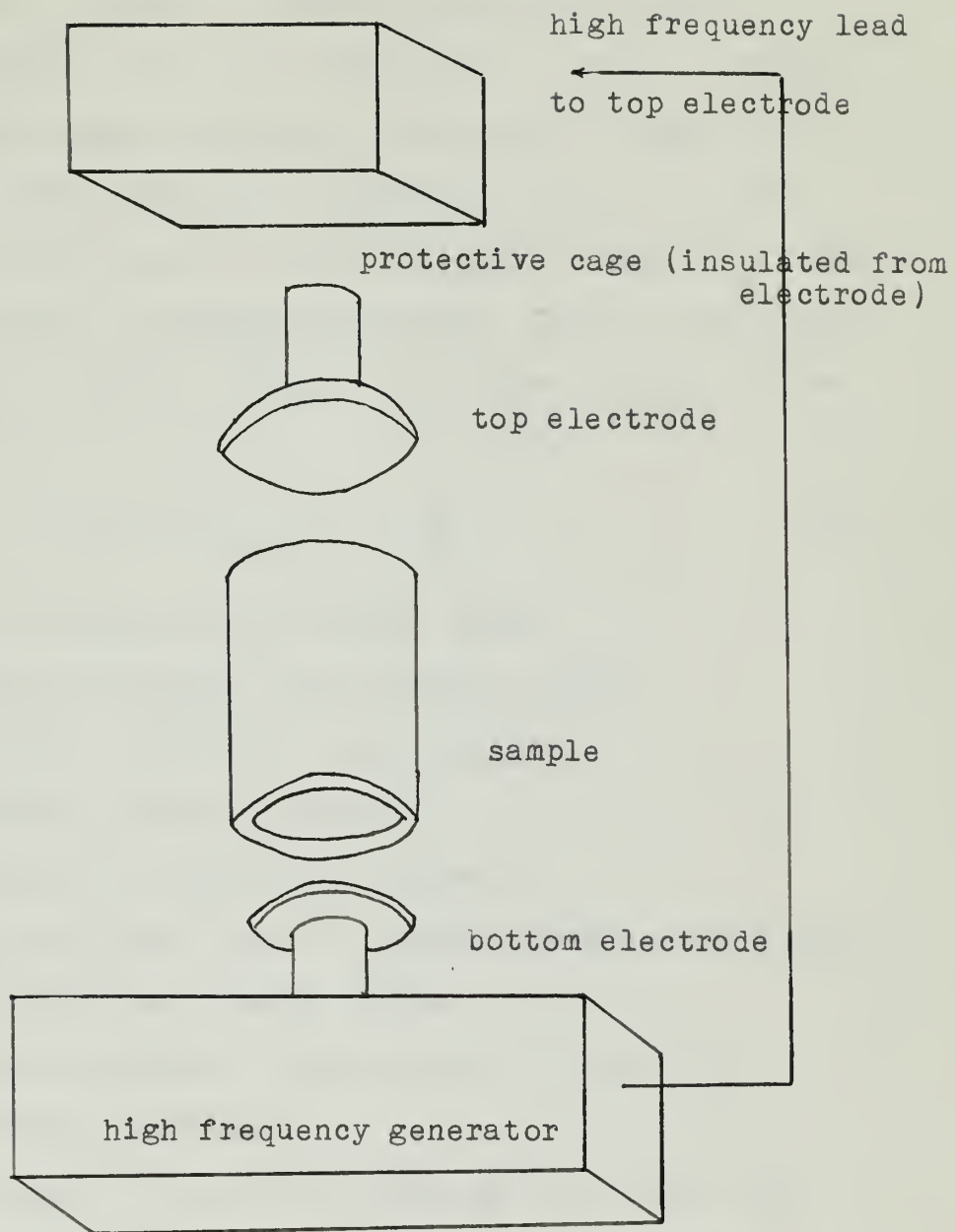
Uniform internal generation of heat may be accomplished by passing an electric current through the sample. The circuit would be a series circuit of generator, sample and ammeters with a voltmeter across the sample. The sample would have to be a conductor or a semi-conductor to permit current flow (3).

The principal advantage of resistance heating is that the power input to the sample can be measured very accurately.

The disadvantages of resistance heating are first, that it is difficult to make good high current connections to the sample. Second, the sample should have a zero temperature coefficient of resistivity. A zero temperature coefficient is necessary to give the sample uniform resistance. These disadvantages led to the consideration of dielectric heating.

Uniform internal generation of heat may be accomplished by dielectric heating of the sample in a high frequency alternating electrical field. The arrangement is diagrammed as follows:





The unit employed in this investigation was a Westinghouse 2 KW dielectric heating unit. The frequency was 27.6 megacycles per second.



The generator supplies high frequency alternating current to the electrodes. The lower electrode is fixed to the generator cover. The upper electrode is adjustable vertically within the protective cage and insulated from it. The cage is hinged to the generator cover. After a sample is placed on the lower electrode, the cage is swung into place and the upper electrode lowered to touch the sample.

A sample should have a high dielectric constant (9) and a high power factor for maximum heating. The heat generated is proportional to their product as is shown in the following formula: (10)

$$P_w = 1.4 \times 10^{-12} V^2 K(\text{p.f.}) f \frac{A}{t} \quad (\text{A})$$

$P_w$  - Rate of generation of heat, watts

$V$  - Voltage developed across sample, volts

$K$  - Dielectric constant, dimensionless

(p.f.) - Power factor, per cent

$f$  - Frequency in cycles per second

$A$  - Cross sectional area of sample, parallel to plane of electrodes, square inches

$t$  - Sample thickness, perpendicular to plane of electrodes, inches

The advantages of dielectric heating are first, that it enables the use of insulators. Many insulators are ceramics (7) (8), which are brittle materials (5), and therefore are of interest. Second, no direct electrical connections are needed to the sample from the electrodes.





A disadvantage is the difficulty of measuring power input to the sample. The protective cage, the electrodes and leads all have capacitance. This stray capacitance forms a parallel circuit with the sample. The effect of the stray capacitance could be cancelled by measuring its value with no load conditions and applying its value to circuit readings taken during load conditions.



## IV

### SELECTION OF MATERIALS

The physical properties of the most suitable material for the tests are combined as follows:

A high modulus of elasticity and a high coefficient of thermal expansion are advantageous. It may be assumed with close approximation, that in this problem a one dimensional state of stress exists. For one dimensional stresses and brittle materials, the stress-strain relationships are linear. Additional advantages are a low value of the ultimate strength and a low coefficient of thermal conductivity.

The combination of these properties can be expressed as follows:

Let  $P$  be the rate of generation of heat necessary to cause rupture. Relating the properties listed above to  $P$  shows that  $P$  varies directly as the ultimate strength and the thermal conductivity, and inversely as the modulus and the thermal expansion.

$$P \approx \frac{\sigma k}{E \alpha}$$

The reciprocal of the constant of proportionality for the given structure was found to be  $1/.0052$ .

Then:

$$P_B = \frac{\sigma k}{E \alpha (.0052)} \quad (B)$$

Two criteria were used in the selection of materials.



Formula B was used to find the required rate of generation of heat. Formula A was then solved for the voltage. 3000 to 5000 volts per inch was the maximum allowable voltage. Higher voltages cause arcing over and high-voltage breakdown.

Numerous materials were investigated. Some were rejected due to excessive power or voltage requirements. Others, such as the carbides, were rejected due to fabrication difficulties, and others due to lack of homogeneity. Many materials could not be completely investigated due to lack of proper data for them. This project desired high dielectric loss materials. Other high frequency work in general has sought low loss materials and high loss materials have been abandoned.

Of the materials investigated, two were finally chosen for consideration. They were Barium Strontium Titanate and Plaster of Paris.

The first sample cylinders were made of Barium Strontium Titanate. The composition and fabrication were composition SB57 as described by E. N. Bunting and colleagues (1), with one addition. Grade AWD Titania was substituted for grade TMO. Grade TMO was not available. One per cent of Antimony Oxide was added to increase the dielectric losses.

The Barium Strontium Titanate looked promising because of its fairly high dielectric constant (2, 4, 6). The dielectric constant was a very critical function of the Titanium Sub-oxide which was formed in firing. The Barium



Strontium Titanate failed to heat when placed in the high frequency field. This is believed to have resulted from incomplete formation of the Sub-oxide having the high dielectric properties.

Plaster of Paris was considered next. It is cheap, easily formed, brittle, has low ultimate strength and is homogeneous. Its principal disadvantage is that it has a low dielectric constant and a low power factor.

The first attempts were with ordinary Plaster of Paris. A brass mandrel was made as a pattern for the inner surface and semi-circular molds for the outside. The molds were made without draft or taper in order to achieve uniform cross section. The lack of draft caused the cylinders to break while being removed from the mandrel. This was overcome by using Hydrocal Plaster of Paris, a high strength grade. The resulting cylinders came off the mandrel readily. They were not usable, however, due to poor dielectric properties.

The next step was to attempt to incorporate additives to increase the dielectric losses. It was felt that the property changes would be small if the percent of additives was small.

Dielectric losses can be increased by the presence of conducting particles. It appeared that graphite would have this effect if it could be uniformly distributed in the material. This was attempted by the use of Aquadag, a colloidal suspension of graphite in water. The Aquadag was mixed with the water and the solution stirred as the Plaster







of Paris was sifted in. The mixture was thoroughly kneaded until the plaster began to set. It was then poured into the mold. The mixture was a thick sludge, even in its least viscous form and did not pour well. The first castings had large voids due to entrapped air. This was eliminated by probing the castings with a stirrer and holding a Burgess Vibro-Tool against the brass mandrel while pouring. The castings remained in the mold for twenty-four hours.

Antimony Oxide was added to the low Aquadag mixtures to further increase the losses. A satisfactory composition was as follows:

Aquadag	5 grams
Antimony Trioxide	8 grams
Water	55 cc
Hydrocal Plaster of Paris	126 grams

The Aquadag, Antimony Trioxide and water were mixed. The Plaster of Paris was sifted into the mixture.

The physical properties for this composition were found to be as follows:

$$E - .59 \times 10^6 \text{ psi}$$

$$\alpha - 10^{-5} \text{ in per in} - ^\circ\text{F}$$

$$k - .25 \text{ Btu per sq ft per hr per ft per } ^\circ\text{F}$$

$$\sigma - 454 \text{ psi}$$

$$K - 6.4 \text{ at } 27.6 \text{ megacycles}$$

$$\text{p.f.} - 1.43\% \text{ at } 27.6 \text{ megacycles}$$

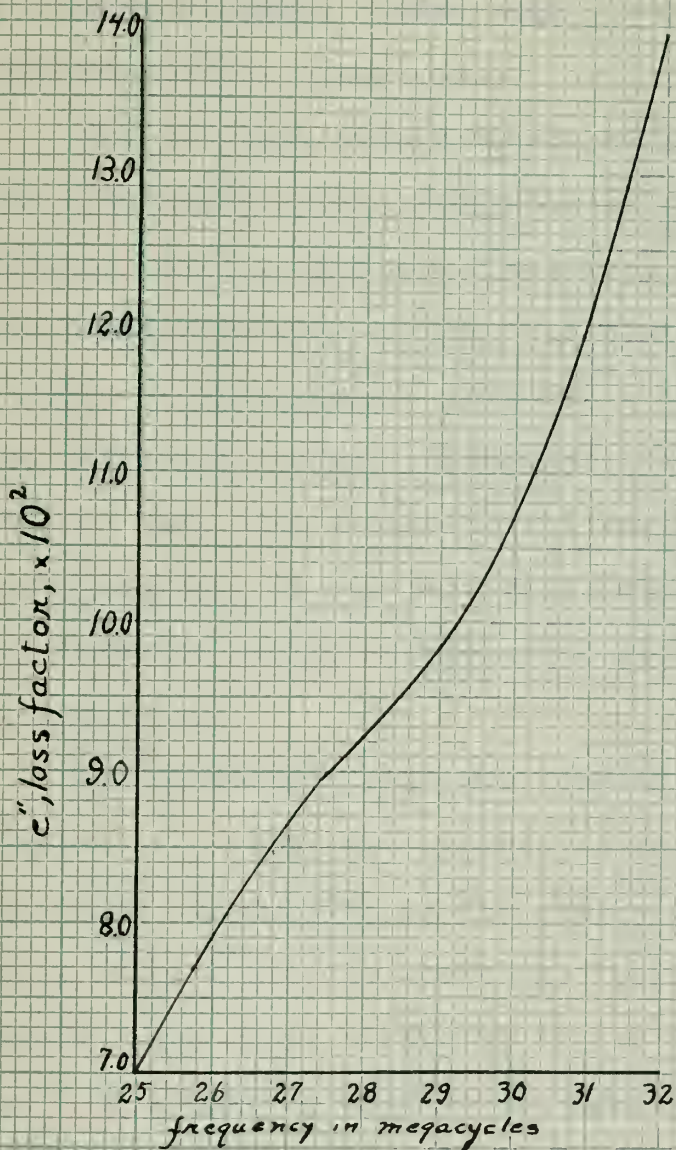
The above mix did not quite fill the mold, so the cylinder was only 3.5 inches long. The cylinder was



uniform and firm without visible air holes. The graphite appeared to be well bonded to the plaster as it would not rub off.



Figure 1



### Loss Factor vs. Frequency

Plaster of Paris with 0.69 wt. per cent Aquadag  
and 4.12 wt. per cent  $Sb_2O_3$





## V

### TESTS CONDUCTED

A range of compositions of Plaster of Paris and Aquadag were heated dielectrically. The ones highest in Aquadag appeared fairly uniform to the eye. Upon heating, agglomerations of graphite became cherry red, showing that the mixture was not homogeneous.

The 3.5 inch sample described above was placed between the electrodes of the generator as shown in the previous sketch. An air hose was fixed in a horizontal position in the protective cage so that the blast impinged on the outer surface of the sample. This was a jury rig but other samples had developed surface temperatures above 190°F and the indications were that this material would develop even higher temperatures. It was felt that unless cooling were provided, the interior of the sample would get excessively hot and disintegrate.

The protective cage was lowered and the generator started. In about ten seconds, a metallic ping was heard. Immediate examination of the sample revealed a very fine crack running axially about one third of the length of the sample. The crack closed up as the sample cooled until it could no longer be seen by the unaided eye. The crack again opened when the sample was reheated. The crack extended through the cylinder wall.

As a check, a 4 inch sample was made with another batch of the same composition. Upon heating, the metallic sound





of rupture was heard but the crack was not visible. The heating was continued and the crack became visible. The power required to cause rupture was estimated at 0.25 KW. The power required as calculated by formula B was 0.28 KW.



## VI

### CONCLUSIONS

The brittle material method appears suitable for design analysis of tubular structures having temperature gradients. The method seems particularly adaptable to the problem of structures having temperature gradients arising from the uniform generation of internal heat.



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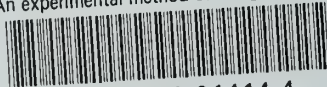
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